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CALCULATION OF LOAD CAPACITY OF GEAR TEETH

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I N D E X

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Calculation of load capacity of gear teeth

Introduction

The load capacity of two meshing gears is limited by three main factors:

1. Bending strength of the teeth.
2. Surface durability of the tooth flanks.
3. Scoring resistance of the tooth flanks

There are two main methods known today to predict strength and durability with good accuracy:

1. Calculation by ISO (International Standard Organisation)
2- Calculation by AGMA (American Gear Manufacturers Association)

The ISO-standards are used predominantly by European gear manufacturers. The formulae given in the MAAG Handbook are based on these.

In the United States gears are calculated by the AGMA standard.

Basically, the two methods are not too much different. However, there are some discrepancies, especially where permissible tooth loads are concerned; e.g. surface-hardened gears are not yet as widely used in the U.S.A. as in Europe. It is probably for this reason that the relation between the allowable stresses for surface-hardened and through-hardened teeth is smaller in the AGMA-standards than in the ISO recommendation.

The main formulae by ISO are listed in this paper. But the discussion of the various factors is outside the scope of this paper; we refer to a publication by VDI (1).

The final ISO-standard has not yet been published. It is expected that it will be available by 1974.

AGMA has introduced the so-called "Service Factor". This factor and its calculation are discussed in this paper.

The third criteria, the calculation of the scoring resistance, has gained in importance in recent years. Especially in surface hardened, high speed/high power gearing it is the scoring limit together with the tooth bending strength which determine the size of the gear.

However, neither AGMA nor ISO have so far provided a formula to predict the tooth loading where scoring must be expected. In this paper a calculation method is described which is based on Prof. Blok's flash temperature theory and which has been in use by MAAG for many years.

1. The formulae by ISO

On pages 5 and 6 the formulae are listed for the calculation of tooth root strength and surface durability. A graph shows the relation between the various stresses and the meaning of the safety factor.

The nominal stress ($\sigma_{F\ ref}$; $\sigma_{H\ ref}$) is based only on the nominal power to be transmitted by the gear, on the tooth size (module) and on the tooth geometry. It is the stress which exists under ideal conditions and can be calculated with good accuracy.

The actual stress (σ_F; σ_H) is arrived at by multiplying the nominal stress by the various K-factors which consider load distribution between the teeth and across the face width, dynamic loads due to tooth errors and overloads inflicted by the driving and driven machines.

The limit stresses ($\sigma_{F\ lim}$; $\sigma_{H\ lim}$) have been found by fatigue testing of various materials, through-hardened and surface-hardened in different ways. The bending fatigue strength of a standard tooth form e.g. has been measured on the pulsating machine. The durability or resistance against pitting has been established by testing actual gears in a back-to-back test arrangement or by roller tests etc.

The max. allowable stresses ($\sigma_{FP\ max}$; $\sigma_{HP\ max}$) are based on the limit stresses corrected by various factors which have an influence on the stress limits. In tooth bending the size factor K_{FX} must be considered for cases where the module is considerably larger than the module of the test gear. Similar factors are introduced in determining $\sigma_{HP\ max}$: The lube oil has an influence, also the peripheral speed at the pitch circle, the surface roughness of the tooth flanks and the hardness ratio of the pinion and gear material.

The safety factor is simply derived by dividing the max. allowable stress by the actual stress.

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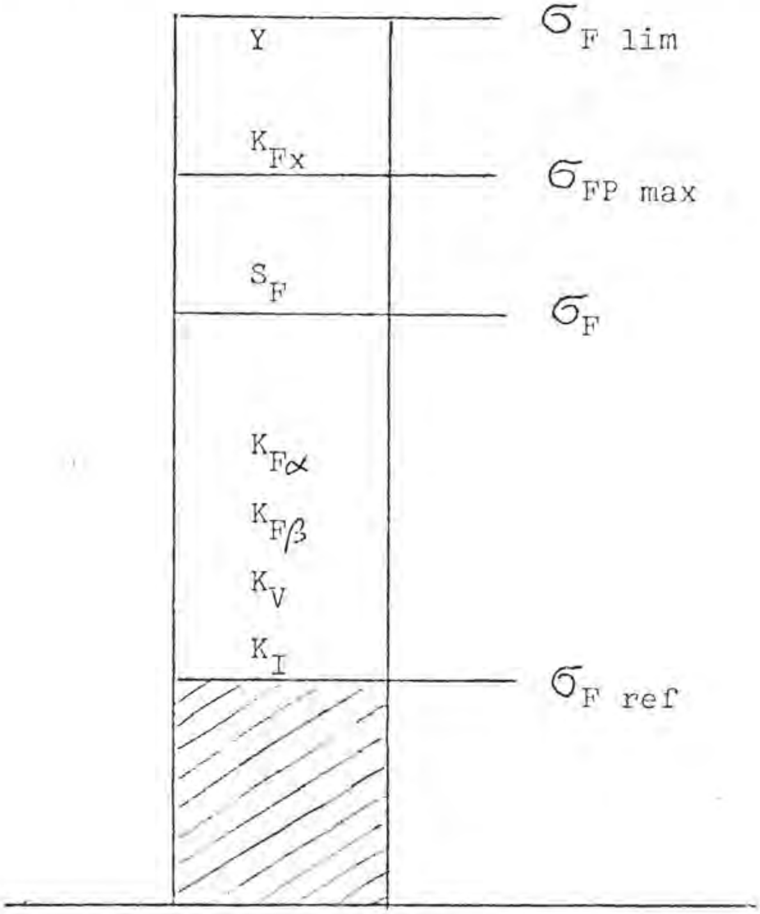
1.1 Calculation of tooth root strength (ISO-Proposal)



Nominal tooth root stress:

$$\sigma_{F\,ref} = \frac{w_t}{m_n} \cdot Y_F \cdot Y_\varepsilon \cdot Y_\beta \cdot Y_S$$

Actual tooth root stress:

$$\sigma_F = \sigma_{F\,ref} \cdot K_I \cdot K_V \cdot K_{F\alpha} \cdot K_{F\beta}$$

Max. allowable tooth root stress:

$$\sigma_{FP\,max} = \sigma_{F\,lim} \cdot K_{FX}$$

Safety factor : (tooth fracture)

$$S_F = \frac{\sigma_{FP\,max}}{\sigma_F}$$

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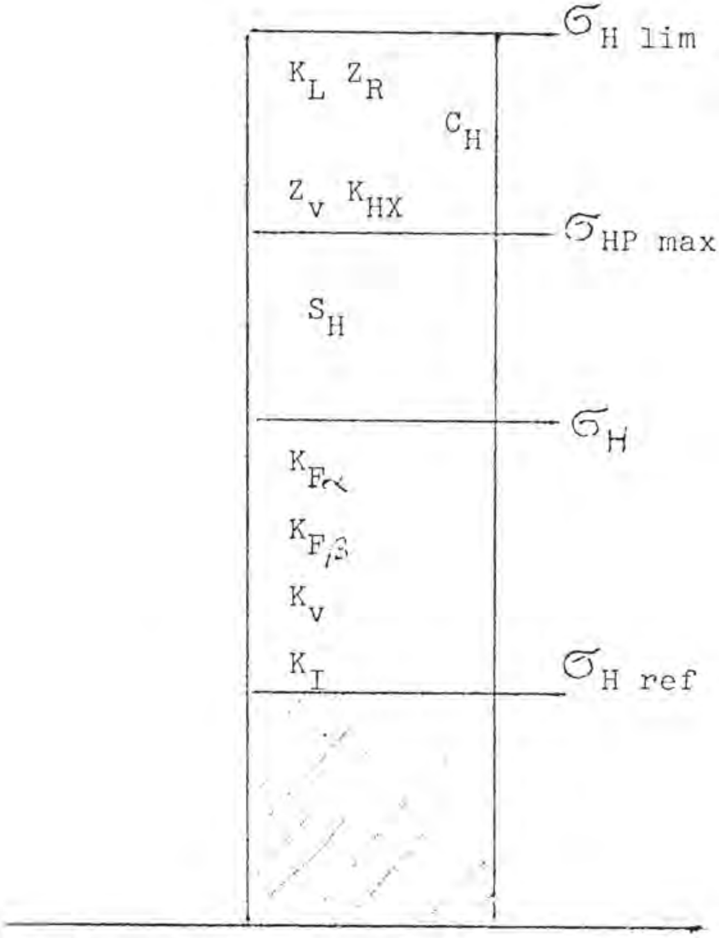
1.2 Calculation of surface durability

(Hertzian stresses) (ISO-Proposal)



Nominal tooth surface stress:

$$\sigma_{H\ ref} = \sqrt{\frac{w_t}{d1} \cdot \frac{u+1}{u}} \cdot Z_H \cdot Z_M \cdot Z_\varepsilon$$

Actual tooth surface stress:

$$\sigma_H = \sigma_{H\ ref} \cdot \sqrt{K_I \cdot K_v \cdot K_{H\alpha} \cdot K_{H\beta}}$$

Max. allowable tooth surface stress:

$$\sigma_{HP\ max} = \sigma_{H\ lim} \cdot K_L \cdot K_{HX} \cdot Z_v \cdot Z_R \cdot C_H$$

Safety factor: (Pitting)

$$S = \sigma_{HP\ max}$$

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S y m b o l s:

Load	w_t	kp/mm	Tooth load per mm of face width
	m_n		Normal module
	d_1	mm	Pinion pitch diameter
	u		Speed ratio
Material	Y_F		Tooth form factor; MAAG-Hb page 123
	$Y_\varepsilon = 1/\varepsilon_\alpha$		$\varepsilon_\alpha =$ Contact ratio
	Y_β		Helix factor
	Y_S		Stress concentration factor
	Z_H		Tooth flank form factor
	Z_M		Material factor
	Z_ε		Contact ratio factor
	$K_{F\beta}$ $K_{H\beta}$		Longitudinal load distribution factor
	K_I		Overload factor
	K_v		Dynamic factor
	$K_{F\alpha}$ $K_{H\alpha}$		Transverse load distribution factor
	K_{FX} K_{HX}		Size factor
	K_L		Lubrication factor
	Z_v		Speed factor
	Z_R, C_R		Surface roughness factor
	C_H		Hardness ratio factor
	C_η		Lube oil factor (additives)
	C_L		Lube oil factor (viscosity)
	C_S		Surface treatment factor
temp.- limits	$\sigma_{F\ lim}$	kp/mm^2	Tooth strength, bending
	$\sigma_{H\ lim}$	kp/mm^2	Surface durability
	$T_{B\ lim}$	$^\circ C$	Flash temperature limit
factors	S_F		Safety factor, tooth bending
	S_H		Safety factor, tooth surface
	S_B		Safety factor, scoring

2. The MAAG method to predict the scoring resistance

The formulae are based on the theory by Prof. Blok on the flash temperature which occurs at the contact points of two tooth flanks when going through mesh. This temperature is a criteria in predicting the danger of scoring (2).

The highest temperatures occur at the tips of all the pinion and gear teeth where the sliding speed is largest. Experience also shows that it is at these points where scoring takes place first. If the flash temperature exceeds a certain level the oil film between the tooth flanks evaporates and looses its capability to carry load. Therefore the factor "lube oil" has a big influence on the scoring limit.

As a point of reference the limit flash temperature has been determined from experience and tests for a straight mineral oil without EP-additives, having a viscosity of $4^{\circ}E$ / 50° C (30 cst / 50° C).
Furthermore, this limit temperature is valid for newly ground gears with a surface roughness of ~1 μm (25 μinch).

$$T_{B\ lim} = 140^{\circ}\ C$$

If for a particular case any of the above factors differ from the values given, the limit flash temperature $T_{B\ lim}$ is corrected accordingly by the respective C-factor to obtain the max. allowable flash temperature $T_{BP\ max}$.

Calculation of the nominal flash temperature $T_{B\ ref}$:

At the tooth tip of the pinion	$T_{B\ ref\ 1} = A \cdot f_1$
At the tooth tip of the gear	$T_{B\ ref\ 2} = A \cdot f_2$

$$A = \frac{30 \cdot w_t}{\varepsilon_\alpha \cdot \cos\alpha_{tw}} \cdot \sqrt[4]{\frac{v}{a^2 \cdot \sin\alpha_{tw}}}$$

w_t	kg/mm	Tooth load per mm of face width
ε_α		Contact ratio
α_{tw}		Working pressure angle (transverse)
v	m/s	Pitch line velocity
a	mm	Centre distance

The factors f_1 and f_2 are taken from the graphs Fig. 3 and 4. They are plotted as a function of the speed ratio u and a factor k_1 and k_2 resp.,

$$k_1 = \left(\frac{u+1}{u}\right) \cdot \left(1 - \frac{tg\,\alpha_{tw}}{tg\,\alpha_{a1}}\right) \text{ whereby: } \cos\alpha_{a1} = \frac{d_{b1}}{d_{a1}}$$

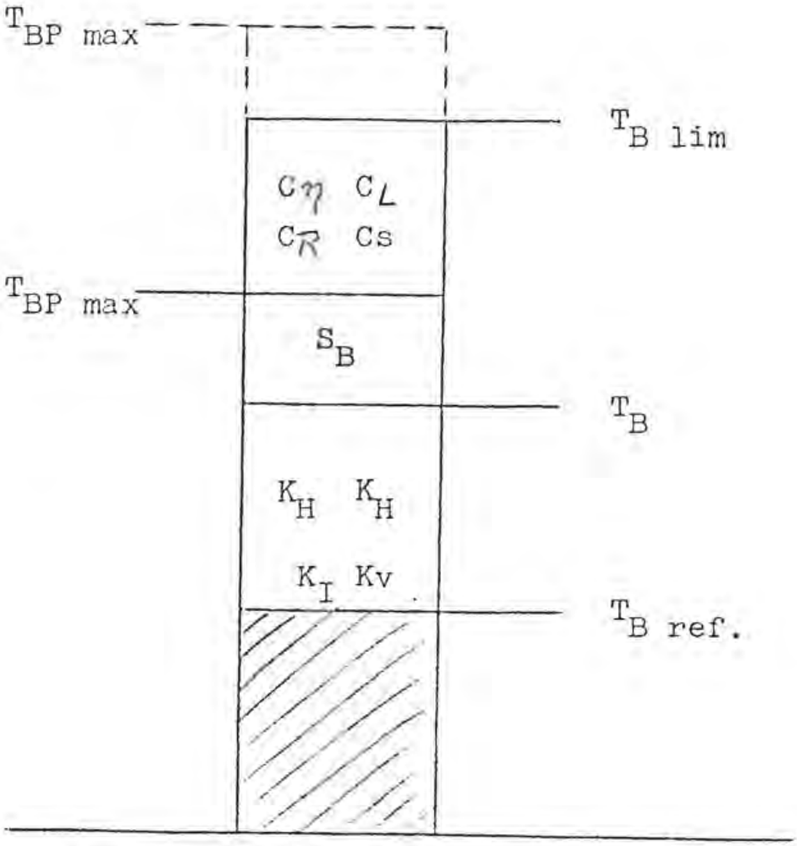
$$k_2 = \left(u+1\right) \cdot \left(1 - \frac{tg\,\alpha_{tw}}{tg\,\alpha_{a2}}\right) \text{ whereby: } \cos\alpha_{a2} = \frac{d_{b2}}{d_{a2}}$$

see fig. 2

The actual flash temperature is obtained by multiplying the nominal flash temperatures $T_{Bref\ 1}$ and $T_{Bref\ 2}$ with the same K-factors which were applied to calculate the actual surface stress.

Similar to the strength and durability calculations, we can derive a <u>Safety Factor</u> S_B for scoring:

(Blok/MAAG-flash temp. theory)



Nominal flash temperature:

$$\underline{T_{Bref} = A \cdot f_1 \ \text{Pinion Tooth-Tip} = A \cdot f_2 \ \text{Gear Tooth-tip}}$$

Actual flash temperature:

$$\underline{T_B = T_{B\ ref} \cdot K_I \cdot K_v \cdot K_{H\alpha} \cdot K_{H\beta}}$$

Max. allowable flash temperature:

$$\underline{T_{BP\ max} = T_{B\ lim} \cdot C_{\eta} \cdot C_L \cdot C_R \cdot C_s}$$

Safety factor (scoring)

$$S_B = \frac{T_{BP\ max}}{T_B}$$

Comments:

As already mentioned, the calculation of the scoring limit is relatively new, but from practical experience a good deal is known today in this field. By comparing these results with the calculated figures, we found that the flash temperature theory is sound and a good instrument to predict the scoring limit. Still, the various C-factors listed above are not yet known adequately. Until further facts are available we have abandoned the idea of calculating a Safety factor. For the time being we simply calculate the nominal flash temperature $T_{B\ ref}$ multiplied by the overload factor K_I. For each gear application we know from practical experience what flash temperatures are permissible to assure a sound safety against scoring.

With growing knowledge and experience we shall eventually be able to calculate with good accuracy a safety factor S_B as laid out on page 10.

3. The AGMA-Approach; Service Factor

AGMA has issued two standards for the rating of single and double helical gear teeth:

1) Rating the strength of gear teeth: AGMA 221.02

2) Rating the surface durability of gear teeth: AGMA 211.02

These two standards contain the basic formulae for the calculation of the tooth bending stress and surface stress (Hertzian stress). They also give allowable stresses for various materials and hardness. These standards are generally applicable throughout the gear industry as far as helical teeth are concerned.

For a number of special gear applications standards are available which reflect the individual design practice in that particular field, e.g.

Rolling mill gears:	323.01
Speed reducers and increasers:	420.03
High speed gear units:	421.06
etc.	

These individual design practices are all based on the standards 211.02 and 221.02.

3.1 The Service Factor

The service factor is best described in the AGMA Standard Practice for High Speed Helical and Heringbone Gear Units, AGMA 421.06. It is a well defined factor and relates the

so called "rated horsepower" to "Service horsepower".
For each mesh there are three factors to be calculated:

1. Service factor for tooth bending strength, one for the pinion- and one for the gear-tooth:

$$K_{SF} = \frac{P_{at}}{P_{ST}} = \frac{\text{horsepower rating, strength}}{\text{service horsepower}}$$

2. Service factor for surface strength:

$$C_{SF} = \frac{P_{ac}}{P_{sc}} = \frac{\text{horsepower rating, durability}}{\text{service horsepower}}$$

This factor is the same for pinion and gear because the surface stresses at the contact point of the two flanks are always equal.

The lowest value of the three is the Service Factor of the gear.

The horsepower ratings P_{at} and P_{ac} are the max. allowable powers to be transmitted by the gear, based on tooth strength and durability respectiveley. The strength rating is calculated using the formula of AGMA 221.02, the durability rating is derived from AGMA 211.02. All K- and C-factors are taken as unity except for:

- Load distribution factors K_m and C_m

  These factors are derived from

Strength:	K_m - AGMA 221.02	Table 2
Durability:	C_m - AGMA 211.02	Fig. 4, first red curve

- Dynamic factors K_v and C_v

Strength:	K_v - AGMA 221.02	Fig. 6 curve 2
Durability:	C_v - AGMA 211.02	Fig. 6 curve 3

The horsepower rating depends further on:

Gear dimensions
Speed ratio
Tooth geometry and module
Material and surface hardness

By definition, the service horsepower is equal to the maximum continuous horsepower capacity of the prime mover. Therefore, it would be wrong if a buyer would order a gear for a somewhat higher horsepower than needed, "just to be on the safe side". This safety is taken care off by the service factor! AGMA 421.06, table 3, recommends service factors for various gear applications.

It is important to note that the Service Factor does not include gear tooth accuracy at all! The load distribution factors K_m and C_m only depend on face width, and the dynamic factors K_v and C_v are only a function of peripheral speed. In actual fact, however, both factors are influenced to a great extent by tooth errors! Therefore, the AGMA Service Factor is no criterion at all for the quality of a gear; it only gives an indication on the dimensions and the material strength of the toothed parts.

References:

(1) Tragfähigkeitsberechnung von Stirn- und Kegelrädern nach DIN 3990 — VDI-Z Band III, 1969, Nr. 4

(2) Lubrication as a Gear Design Factor — Proceedings of the International Conference on Gearing, London 1958

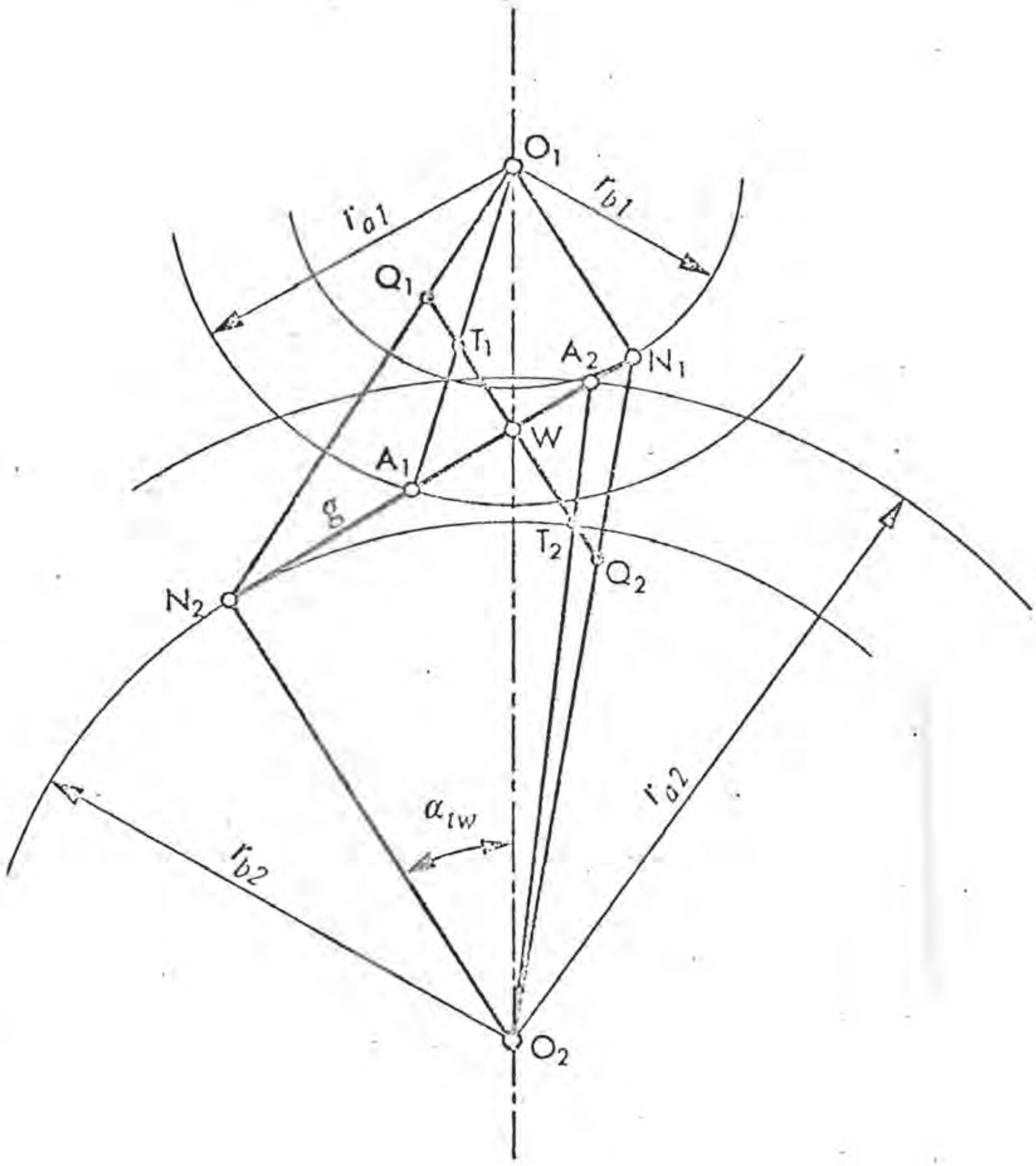
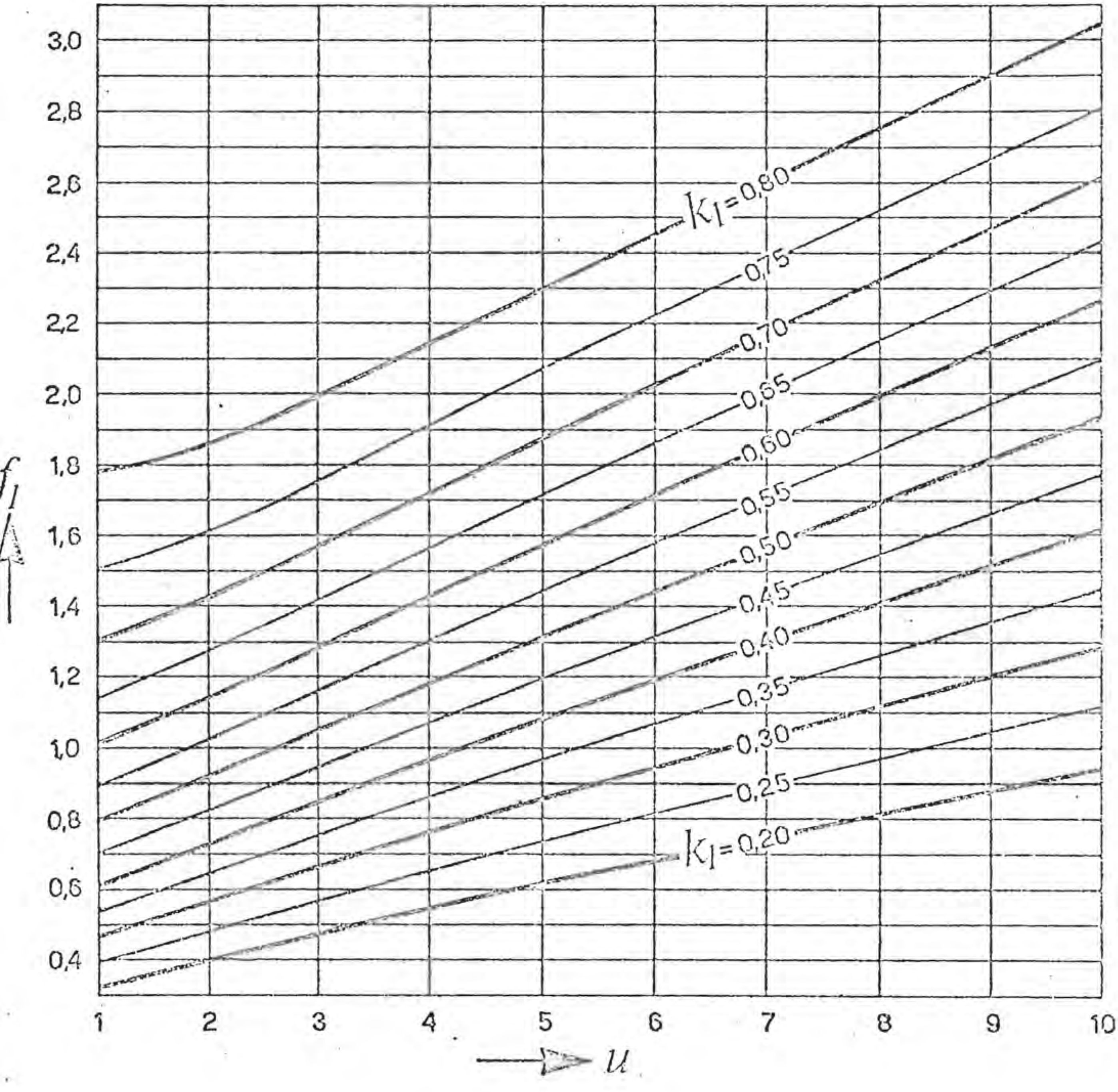


Fig. 2: Tooth contact geometry in involute gears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f_I
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2
3
4
5
6
7
8
9
10
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K_I=0,20

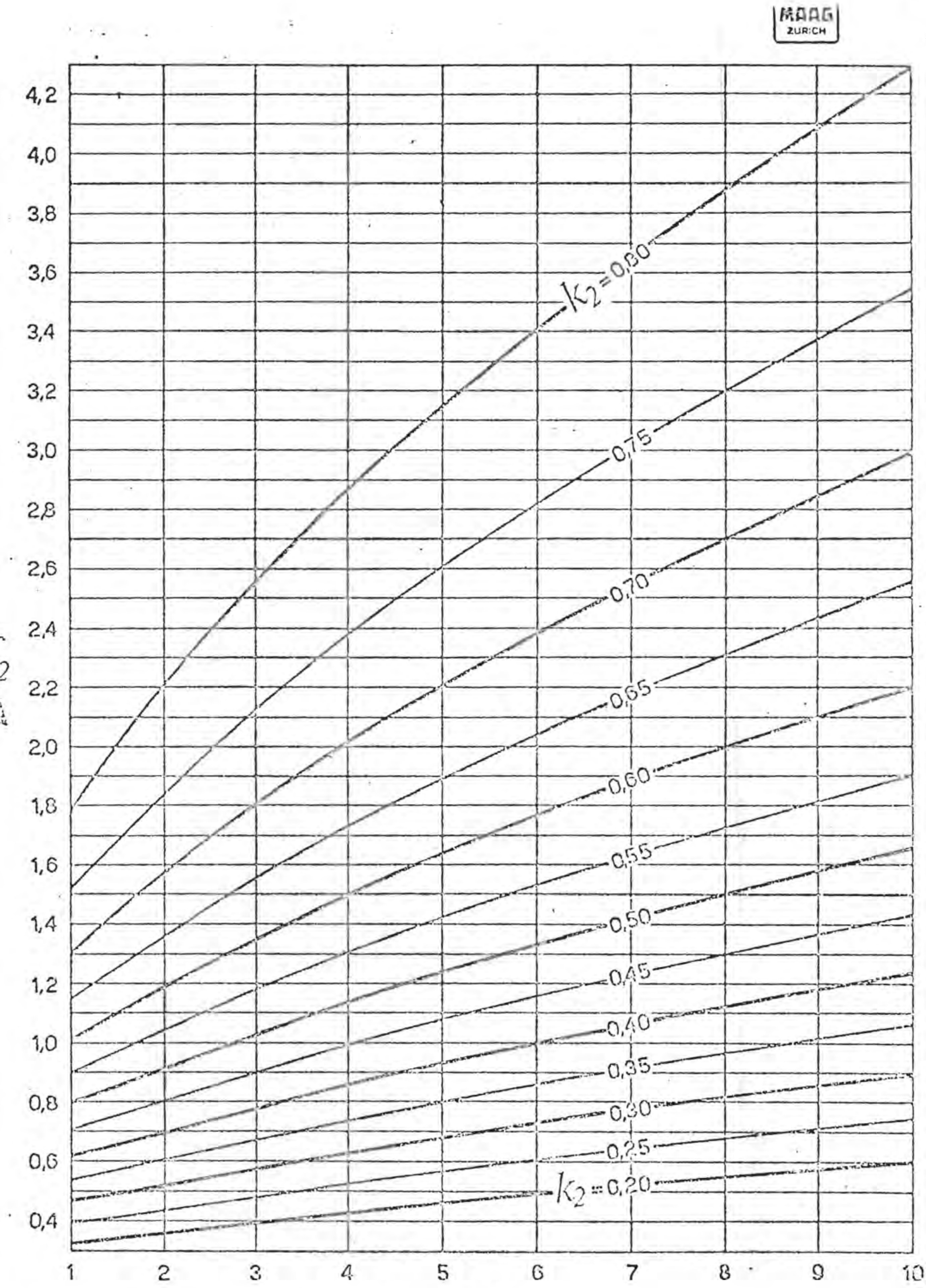

Fig. 4: The function $f_2 = f(u, k_2)$